THE STRUCTURE OF ATOMIC NUCLEI

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ABSTRACT. The possibility of atomic nuclei consisting of smaller nuclei of comparable masses is discussed. The depth and width of the potential holes corresponding to the nuclear energy are calculated for some nuclei. The expected masses and charges of the exchange particles holding the nuclei together are calculated for some cases. The nuclear spins of some of the rare gases are derived by comparison.

The study of the properties of the atomic nucleus has received much attention in recent years both from the theoretical and the experimental standpoints. Prominent among these has been the work of Bohr who has considered in detail a model of the nucleus, now familiar as the liquid drop model. On the other hand, the essence of a theory propounded by the present writer some years ago[1] is to consider the nucleus as resembling a binary or multiple stellar system in which there are a few components of comparable masses. A few simple assumptions led to a unified scheme of arrangement of the different elements, which fell into naturally related groups. The components of the nucleus were assumed to be provided by the rare gas nuclei and the protons. Considering only the binary systems composed of two components, a satisfactory picture was obtained of the building up of chemically similar nuclei. On making a second assumption of positron emission (the newer version of electronic absorption), whole series of chemically similar elements take up their places in a natural way in a new kind of periodic table, which is reproduced here as Table I. The fundamental particles have since increased in number following the discovery of the neutron, the positron, the mesotron, and possibly the neutrino and the antineutrino. A large number of new isotopes has also been discovered, and the laws of interaction have been investigated. The inherent consistency of the original scheme makes it worth our while to revise and re-assess it in the light of these new discoveries.

The large number of isotopes which have been discovered during the past few years fit in well into the scheme for two nuclei made up of two components. A very complete list of the lighter isotopes up to mass 40 has been given by Barkas.[2] The isotopes of Na, Mg, K and Ca, and most of Cl, fit in quite well, isotope by isotope, as constituted of Ne, A and He nuclei and protons. The existence of a number of isotopes of Ba has come to light and these find places in the nuclei originating from X.

TABLE I

<table>
<tr><th></th><th>H</th><th>He</th><th colspan="3">Ne</th><th colspan="3">A</th><th colspan="6">Kr</th><th colspan="9">X</th><th>Rn</th></tr>
<tr><td></td><td></td><td></td><td colspan="3">(18-27)</td><td colspan="3">(35-40)</td><td colspan="6"></td><td colspan="9"></td><td></td></tr>
<tr><td></td><td>1</td><td>4</td><td>20</td><td>21</td><td>22</td><td>36</td><td>38</td><td>40</td><td>78</td><td>80</td><td>82</td><td>83</td><td>84</td><td>86</td><td>124</td><td>126</td><td>128</td><td>129</td><td>130</td><td>131</td><td>132</td><td>134</td><td>136</td><td>222</td></tr>
<tr><td>H</td><td>—</td><td>—</td><td>—</td><td>—</td><td>Na</td><td>Cl</td><td>K</td><td>K</td><td>Br</td><td>Br</td><td>—</td><td>—</td><td>Rb</td><td>Rb</td><td>—</td><td>I</td><td>—</td><td>—</td><td>—</td><td>—</td><td>Cs</td><td>—</td><td>—</td><td>—</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Na (20-29)</td><td colspan="3">Cl (32—40)
K (36—40)</td><td colspan="6"></td><td colspan="9"></td><td></td></tr>
<tr><td>He</td><td>—</td><td>—</td><td>Mg</td><td>Mg</td><td>Mg</td><td>Ca</td><td>Ca</td><td>Ca</td><td>Se</td><td>Sr</td><td>Sr</td><td>Sr</td><td>Sr</td><td>Zr</td><td>Te</td><td>Te</td><td>—</td><td>—</td><td>Ba</td><td>Ba</td><td>Ba</td><td>Ba</td><td>Ce</td><td>Ra</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">Mg 22-31</td><td colspan="3">Ca (38—45)</td><td colspan="6"></td><td colspan="9"></td><td></td></tr>
<tr><td>Ne</td><td>—</td><td>—</td><td colspan="3">—</td><td colspan="3">Atc wts : 56-62 (53-67) Atc No. : 28 and less

Ni, Fe, Co : Wt : 54-62 Nos : 26, 27, 28.</td><td colspan="6">Atc. wts. : 98-108 (96—113) Atc. Nos. : 46 and less.

Ru, Rh, Pd, Mo, Ma :
Atc. wts : 96-108 :
Atc Nos : 42, 43, 44, 45, 46.</td><td colspan="9">Atc. wts 144-158 (142-163) ;
Atc. Nos. : 68 and less.

Atc. wts. of Group I of rare earths, La to Gd ; 139—158
Atc. Nos. of Group I of rare earths : 57-64.</td><td>—</td></tr>
<tr><td>A</td><td>—</td><td>—</td><td colspan="3">—</td><td colspan="3">—</td><td colspan="6">Atc wts. : 114-126 (113-126)
Atc Nos : 54 and less.

Cd, In, Sn, Sb, Te :
Atc. wts. : 106-124 ;
Atc. Nos : 48-52.</td><td colspan="9">Atc wts : 160-176 (159—176) ;
Atc Nos : 72 and less.

Group II of rare earths : Tb to Hf :
Atc. wts : 159-176 :
Atc. Nos. : 65-72.</td><td>—</td></tr>
<tr><td>Kr</td><td>—</td><td>—</td><td colspan="3">—</td><td colspan="3">—</td><td colspan="6">—</td><td colspan="9">Atc wts : 202—222 ;
Atc. Nos. 90 and less.

Radioactive elements :
Atc. wts : 206—238 :
Atc. Nos : 92 and less</td><td>—</td></tr>
<tr><td>X</td><td>—</td><td>—</td><td colspan="3">—</td><td colspan="3">—</td><td colspan="6">—</td><td colspan="9">—</td><td>—</td></tr>
</table>

This table primarily includes the commoner isotopes [4]. Figures in brackets indicate inclusion of all the isotopes of Ne, A and the synthesised elements about whose existence there is any evidence

Coming to the question of the interaction forces holding the nucleus together, we cannot do better than follow the lead of the well-determined investigations into the structure of the deuteron. The nature of the force there found is of the exchange type, first introduced by Majorana. We may imagine the deuteron nucleus as composed of two neutrons with a positron moving between them, attaching itself to the neutrons alternately. During the period of such an attachment the neutron becomes a proton, and the nucleus consists of a neutron and a proton. When the positron disengages itself from the proton and goes over to the neutron, the nucleus once again consists of a proton and a neutron, with their positions virtually interchanged. This mechanism may or may not correspond to reality in nature; but the force derived from this idea of the interchange of positions of the constituents of the nucleus has been shown to be sufficient to hold the deuteron nucleus together. The potential function is found to be of the form $V\epsilon^{-\lambda r}/r$, where $1/\lambda$ defines the range of the internuclear force and is given by

$$\lambda = \frac{2\pi Mc}{h},$$

where M is the mass of the exchange particle.[3] This range is taken extremely small, the exchange force being effective for values smaller than this and virtually disappearing at greater distances. The depth, V, of the potential hole, and its breadth have been investigated for certain representative forms of the function.[4] To calculate the approximate values of the depth and breadth of the potential hole of the deuteron nucleus, we follow the general procedure adopted by Bethe and Bacher.[5] The wave equation for a nucleus made up of two nuclei of masses M_1 and M_2 can be written down as

$$\frac{1}{M_1}\left(\frac{\partial^2\psi}{\partial x_1^2}+\frac{\partial^2\psi}{\partial y_1^2}+\frac{\partial^2\psi}{\partial z_1^2}\right)+\frac{1}{M_2}\left(\frac{\partial^2\psi}{\partial x_2^2}+\frac{\partial^2\psi}{\partial y_2^2}+\frac{\partial^2\psi}{\partial z_2^2}\right)+\frac{8\pi^2}{h^2}E_0\psi$$
$$=\frac{8\pi^2}{h^2}.J(r).[\psi]. \qquad (1)$$

Here ψ signifies a function of the co-ordinates, as

$$\psi \equiv \psi(x_1, y_1, z_1, s_1;\ x_2, y_2, z_2, s_2); \qquad \ldots \quad (2a)$$

and $[\psi]$ the same function with positional co-ordinates interchanged, as

$$[\psi] \equiv \psi(x_2, y_2, z_2, s_1;\ x_1, y_1, z_1, s_2). \qquad \ldots \quad (2b)$$

Here x_1, y_1, z_1, s_1 denote the co-ordinates of position and the spin of M_1, and x_2, y_2, z_2, s_2, similar quantities for M_2. The equation indicates an exchange force of the Majorana type, only the spatial co-ordinates of M_1 and M_2 being interchanged without their spins being affected. The condition for this is that

$$\psi(x_2, y_2, z_2, s_1;\ x_1, y_1, z_1, s_2) = F_1(x_2, y_2, z_2;\ x_1, y_1, z_1).F_2(s\ ,\ s_2) \quad \ldots \quad (3)$$

and

$$\psi(x_1, y_1, z_1, s_1;\ x_2, y_2, z_2, s_2) = F_1(x_1, y_1, z_1;\ x_2, y_2, z_2).F_2(s_1, s_2). \quad \ldots \quad (4)$$

This reduces (1) to

$$\frac{1}{M_1}\Delta_{x_1}F_1(x_1, x_2)+\frac{1}{M_2}\Delta_{x_2}F_1(x_1, x_2)+\frac{8\pi^2}{h^2}E_0F_1(x_1, x_2)$$

$$=\frac{8\pi^2}{h^2}J(r)\,F_1(x_2, x_1). \quad \ldots \quad (5)$$

Here x_1 and x_2 denote, in short, all the co-ordinates x_1, y_1, z_1, and x_2, y_2, z_2. $J(r)$ is the potential energy. We introduce new variables x, y, z, describing the motion of the centre of mass of the system, and r, θ, ϕ, the polar co-ordinates of M_2 relative to M_1. The relation between the different co-ordinates is given by

$$x=\frac{M_1x_1+M_2x_2}{M_1+M_2}, \quad \ldots \quad (6)$$

$$y=\frac{M_1y_1+M_2y_2}{M_1+M_2}; \quad \ldots \quad (7)$$

$$z=\frac{M_1z_1+M_2z_2}{M_1+M_2}; \quad \ldots \quad (8)$$

$$r\sin\theta\cos\phi=x_2-x_1; \quad \ldots \quad (9)$$

$$r\sin\theta\sin\phi=y_2-y_1; \quad \ldots \quad (10)$$

$$r\cos\theta=z_2-z_1. \quad \ldots \quad (11)$$

The introduction of these co-ordinates reduces (5) to

$$\frac{1}{M_1+M_2}\left(\frac{\partial^2F_1}{\partial x^2}+\frac{\partial^2F_1}{\partial y^2}+\frac{\partial^2F_1}{\partial z^2}\right)+\frac{2}{M}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial F_1}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2F_1}{\partial\phi^2}\right.$$

$$\left.+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial F_1}{\partial\theta}\right)\right\}+\frac{8\pi^2}{h^2}E_0\psi=\frac{8\pi^2}{h^2}.J(r).F_1(x, y, z, -r) \quad \ldots \quad (12)$$

Here $M/2$ is the reduced mass of the system given by

$$M=\frac{2M_1M_2}{M_1+M_2}. \quad \ldots \quad (13)$$

We separate the equations by expressing F_1 as the product of a function of x, y, z, and a function of r, θ, ϕ.

$$F_1(x, y, z, r, \theta, \phi)=F(x, y, z).U(r, \theta, \phi). \quad \ldots \quad (14)$$

Introducing this in (12), it separates out into the two equations

$$\frac{\partial^2F}{\partial x^2}+\frac{\partial^2F}{\partial y^2}+\frac{\partial^2F}{\partial z^2}+\frac{8\pi^2}{h^2}(M_1+M_2)E_1.F=0, \quad \ldots \quad (15)$$

and
$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2U}{\partial\phi^2}+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial U}{\partial\theta}\right)$$

$$+\frac{4\pi^2M}{h^2}E.\,U(r)=\frac{4\pi^2}{h^2}M.\,J(r).\,U(-r)=0; \quad \ldots \quad (16)$$

with $$E_1 + E = E_0. \quad \dots (17)$$
The equation (16) can be expressed as

$$\frac{h^2}{4\pi^2 M} . \Delta U(r) + E.\ U(r) = J(r).\ U(-r). \quad \dots (18)$$

Here

$$-E = \epsilon, \quad \dots (19)$$

the binding energy of M_1 and M_2.

Assuming the potential energy to be spherically symmetrical, we can separate (18) into the polar coordinates; as

$$U(r) = \frac{U_l(r)}{r} .\ P_{lm}(\theta).e^{im\phi} \quad \dots (20)$$

P_{lm} being a spherical harmonic duly normalized. Now, if the polar coordinates of a point x, y, z, are r, θ, ϕ, those of the point $-x, -y, -z$, will be $r, \pi-\theta, \pi+\phi$. Also,

$$P_{lm}(\pi-\theta)e^{im(\pi+\phi)} = (-1)^l . P_{lm}(\theta)e^{im\phi} . \quad (21)$$

The wave-equation for u becomes

$$\frac{h^2}{4\pi^2 M}\left(\frac{\partial^2 u_l}{\partial r^2} - \frac{l(l+1)}{r^2} u_l \right) + E u_l = (-1)^l .\ J(r).u_l. \quad (22)$$

For the ground state, $l = 0$, and assuming J to be negative,

$$-\frac{h^2}{4\pi^2 M} \frac{d^2 u}{dr^2} = \{J(r) - E\}.u, \quad (23)$$

u is not to become infinite for large values of r, and is to vanish as r for small values of r. $J(r)$ has the essential character of an exchange force, being effective within a range a, say, and becoming negligible in comparison with E for $r > a$.

We have assumed the exchange interaction potential to be of the form

$J(r) = -V.\frac{e^{-\lambda r}}{r}$, the range $\frac{1}{\lambda}$ being defined by

$$\lambda = \frac{2\pi M' c}{h}, \quad \dots (24)$$

M' being the mass of the exchange particle. With

$$J(r) = -V.\frac{e^{-\lambda r}}{r}, \quad \dots (25)$$

equation (23) becomes

$$\frac{d^2 u}{dr^2} = \frac{4\pi^2 M}{h^2}\left(-V.\frac{e^{-\lambda r}}{r} - E \right) u. \quad \dots (26)$$

If we put
$$x=\lambda r, \qquad \dots (27)$$
then,
$$\lambda=\frac{dx}{dr}$$
and
$$\frac{d}{dr}\left(\frac{du}{dr}\right)=\frac{d}{dr}\left(\frac{dx}{dr}\frac{du}{dx}\right)=\frac{dx}{dr}\cdot\frac{d}{dx}\left(\frac{dx}{dr}\frac{du}{dx}\right)=\lambda^2\frac{d^2u}{dx^2}. \qquad \dots (28)$$
Equation (26) becomes
$$\lambda^2\frac{d^2u}{dx^2}=-\frac{4\pi^2M}{h^2}\left(\lambda.\,V\frac{e^{-x}}{x}+E\right)u. \qquad \dots (29)$$
Or,
$$\frac{d^2u}{dx^2}+\frac{4\pi^2MV}{h^2\lambda}\cdot\frac{e^{-x}}{x}\cdot u-\frac{4\pi^2M\epsilon}{h^2\lambda^2}u=0 \qquad \dots (30)$$
This can be put in the form
$$\frac{d^2u}{dx^2}+a\left(\frac{e^{-x}}{x}-b\right)u=0 \qquad \dots (31)$$
with
$$a=\frac{4\pi^2MV}{h^2\lambda}, \qquad \dots (32)$$
and
$$b=\frac{\epsilon}{V\lambda}. \qquad \dots (33)$$
At $x=0$, (31) assumes the form
$$\frac{d^2u_0}{dx^2}+\frac{a}{x}.u_0=0. \qquad \dots (34)$$
The function
$$u_0=A.x.e^{-\frac{a}{2}x} \qquad \dots (35)$$
satisfies this relation, because
$$\frac{d^2u_0}{dx^2}=-\frac{au_0}{x}+\frac{a^2}{4}u_0; \qquad \dots (36)$$
which, for $x\longrightarrow 0$, becomes
$$\frac{d^2u_0}{dx^2}+\frac{au_0}{x}=0. \qquad \dots (37)$$
Following the procedure adopted in similar cases,[6] we put the solution to be
$$u=u_0v \qquad \dots (38)$$

where v is developable in a series. On substituting in (31),

$$v'' + v'\left(\frac{2}{x} - a\right) + av\left(\frac{e^{-x}}{x} - \frac{1}{x} + \frac{a}{4} - b\right) = 0 \qquad \text{.. (39)}$$

or

$$\begin{aligned} v'' + v'\left(\frac{2}{x} - a\right) + av\left(\frac{a}{4} - b - 1 + \frac{x}{2!} - \frac{x^2}{3!} + \ldots\ldots\right) \\ \equiv v'' + p(x).v' + q(x)v \\ = 0\,; \end{aligned} \qquad \text{... (40)}$$

where $p(x)$ and $q(x)$ are the coefficients of v' and v. Here $p(x)$ has a pole of the first order at $x=0$, and $q(x)$ has no pole of an order higher than the second. We put[7]

$$\begin{aligned} L(v) \equiv x^2v'' + x(2-ax)v' + v\left(\frac{a^2}{4}x^2 - abx^2 - ax^2 + \frac{ax^3}{2!} - \frac{ax^4}{3!} + \ldots\ldots\right) \\ \equiv x^2v'' + v'P_1(x) + vP_2(x) \\ = 0. \end{aligned} \qquad \text{... (41)}$$

Here, let

$$P_1(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots \qquad \text{... (42)}$$

and

$$P_2(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \ldots \qquad \text{... (43)}$$

Comparing (42) and (43) with (41),

$$\alpha_0 = 2,$$

$$\alpha_1 = -a,$$

and

$$\alpha_n = 0 \qquad (\text{for } n > 1) \qquad \text{... (44)}$$

and

$$\beta_0 = 0,$$

$$\beta_1 = 0,$$

$$\beta_2 = \frac{a^2}{4} - ab - a,$$

$$\beta_3 = \frac{a}{2!}$$

$$\beta_4 = -\frac{a}{3!}, \text{ etc.} \qquad \text{.. (45)}$$

We express v as the series

$$v = x^\rho(c_0 + c_1x + c_2x^2 + \ldots). \qquad \text{.. (46)}$$

In general, with

$$L(x) = x^\rho(f_0 + f_1.x + f_2.x^2 + \ldots) \qquad \text{... (47)}$$

the coefficients c's are obtained by making the individual coefficients in (47) vanish ; when we get

$$f_0(\rho)=\rho(\rho-1)+\rho.\alpha_0+\beta_0,$$

$$f_1(\rho)=\rho.\alpha_1+\beta_1$$

$$f_n(\rho)=\rho.\alpha_n+\beta_n. \qquad \dots \quad (48)$$

In our case

$$f_0(\rho)=\rho(\rho-1)+2\rho$$

$$f_1(\rho)=-a\rho$$

$$f_2(\rho)=a\left(\frac{a}{4}-b-1\right)$$

$$f_3(\rho)=\frac{a}{2\,!}$$

$$f_4(\rho)=-\frac{a}{3\,!}, \text{ etc.} \qquad \dots \quad (49)$$

The coefficients c's satisfy the equations

$$c_0.f_0(\rho)=0, \qquad \dots \quad (50)$$

$$c_1.f_0(\rho+1)+c_0.f_1(\rho)=0, \qquad \dots \quad (51)$$

$$c_2.f_0(\rho+2)+c_1.f_1(\rho+1)+c_0.f_2(\rho)=0, \qquad \dots \quad (52)$$

etc.

The first of these gives the indicial equation ; for,

$$\rho(\rho-1)+2\rho=0 \qquad \dots \quad (53)$$

giving the values

$$\rho=0$$

and

$$\rho=-1.$$

For $\rho=0$, (51) gives

$$c_1=0\,; \qquad \dots \quad (54)$$

$$c_2=-\tfrac{1}{6}aB\,; \qquad \dots \quad (55)$$

where

$$B=\frac{a}{4}-b-1\,; \qquad \dots \quad (56)$$

$$c_3=-\frac{1}{12}c_0.a\left(\frac{aB}{3}+\frac{1}{2}\right); \qquad \dots \quad (57)$$

$$c_4=\frac{c_0.a}{120}\left\{1+aB^2-\frac{a^2B}{2}-\frac{3a}{4}\right\}, \qquad \dots \quad (58)$$

etc.

For $\rho=-1$, we get identical coefficients for the series in v, which is to be expected as the two values of ρ differ by unity. A second solution would be of the form

$$v = g.v_1(x).\log x + v_2(x), \qquad \dots \quad (59)$$

where

$$v_1 = c_0 + c_1 x + c_2 x^2 + \dots \qquad \dots \quad (60)$$

and

$$v_2 = c'_0 + c'_1 x + c'_2 x^2 + \dots \qquad \dots \quad (61)$$

The solution (59) contains the term $\log x$; and, therefore, it is of no interest to us.[6]

Thus we take

$$v = A.v_1(x). \qquad \dots \quad (62)$$

From this,

$$u = A.x.e^{-\frac{ax}{2}}.v = A.x.e^{-\frac{ax}{2}}(c_0 + c_2 x^2 + c_3 x^3 + \dots) \qquad \dots \quad (63)$$

and

$$u' = A.e^{-\frac{ax}{2}}(c_0 + c_2 x^2 + c_3 x^3 + \dots) - \frac{A.a}{2}x\, e^{-\frac{ax}{2}}(c_0 + c_2 x^2 + c_3 x^2 + \quad)$$

$$+ A.x.e^{-\frac{ax}{2}}(2c_2 x + 3c_3 x^3 + 4c_4 x^3 + \dots)$$

$$= A.x.e^{-\frac{ax}{2}}\left\{\frac{c_0}{x} - \frac{ac_0}{2} + 3c_2 x + \left(4c_3 - \frac{ac_2}{2}\right)x^2 + \left(5c_4 - \frac{ac_3}{2}\right)x^3 + \dots \right. \qquad (64)$$

Putting u and u' in the form

$$u = A.c_0.x.e^{-\frac{ax}{2}}(1 + b_1 x + b_2 x^2 + \dots) \qquad \dots \quad (65)$$

and

$$u' = A.c_0.x.e^{-\frac{ax}{2}}\left(\frac{1}{x} - a_0 + a_1 x - a_2 x^2 + \dots\right), \qquad \dots \quad (66)$$

$$\frac{u'}{u} = \frac{1}{x} - a_0 + (a_1 - b_2)x + (a_0 b_2 - a_2)x^2 + \dots$$

$$= \frac{1}{x} - \frac{a}{2} + \frac{ax}{3}\left(1 + b - \frac{a}{4}\right) + \frac{ax^2}{3}\left(-\frac{a^2}{12} + \frac{ab}{3} + \frac{a}{3} - \frac{1}{2}\right) + \dots \qquad \dots \quad (67)$$

Following Bethe and Backer, we can derive some useful information from (67). For, $J(r)$ or V can be assumed to be negligible for $r > 1/\lambda$; *i.e.*, $x > 1$. Then (30) reduces to

$$\frac{d^2u}{dx^2} = \frac{4\pi^2 M\epsilon}{h^2\lambda^2}.x. \qquad \dots \quad (68)$$

Or

$$u = c.e^{-\frac{2\pi\sqrt{M\epsilon}}{h\lambda}.x} \qquad \dots \quad (69)$$

and

$$u' = -\frac{2\pi c\sqrt{M\epsilon}}{h\lambda}.e^{-\frac{2\pi\sqrt{M\epsilon}}{h\lambda}.x}. \qquad \dots \quad (70)$$

so that

$$\frac{u'}{u} = -\frac{2\pi\sqrt{M\epsilon}}{h\lambda}. \qquad \dots \quad (71)$$

The shape of the u—x curve is as shown in Fig. 1.

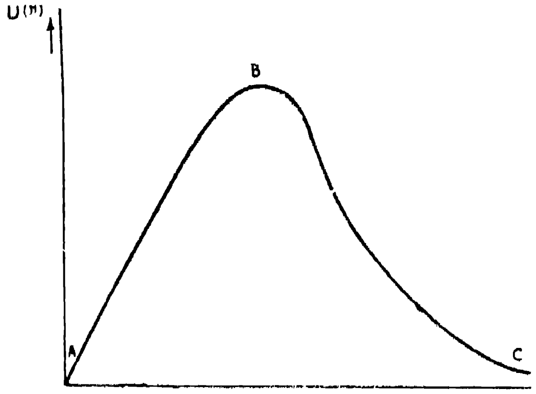

FIG. 1

If we evaluate u'/u for any point, and if it is found to be positive, the point must lie on the ascending part of the curve, AB. Similarly, a point for which u'/u is negative lies on the descending part BC of the u—x curve. The former condition holds for too low a value of $J(r)$ and indicates instability; while, on the other hand, the latter condition denotes stability. We may suppose the values of u'/u calculated for $x > 1$ and for $x < 1$ to join up smoothly at $x=1$.

To a first approximation, from (67)

$$\frac{u'}{u} = 1 - \frac{a}{2}. \qquad \ldots (72)$$

Equating this to (71)

$$1 - \frac{a}{2} = -\frac{2\pi\sqrt{M\epsilon}}{h\lambda}. \qquad \ldots (73)$$

Substituting the values $M = 1.67 \times 10^{-24}$ gm., $\epsilon = 3.42 \times 10^{-6}$ erg., $M' = 130 \times$ the electronic mass, we find for the deuteron nucleus,

$$\frac{2\pi\sqrt{M\epsilon}}{h\lambda} = 0.68. \qquad \ldots (74)$$

This gives

$$a = 3.36. \qquad \ldots (75)$$

Retaining terms including x in (67), we get the second approximation,

$$1 - \frac{a}{2} - \frac{a^2}{12} + \frac{ab}{3} + \frac{a}{3} = -0.68. \qquad \ldots (76)$$

From (32) and (33)

$$ab = \frac{4\pi^2 M\epsilon}{h^2\lambda^2} = 0.46. \qquad \ldots (77)$$

This gives

$$a = 3.8. \qquad \ldots (78)$$

Retaining terms up to x^2 in (67), we get the further approximation

$$\frac{u'}{u}=1+\frac{ab}{3}-\frac{a}{3}+\frac{a}{9}\times ab+\frac{a^2}{36}-\frac{a^3}{36}=-0.68. \qquad \ldots \quad (79)$$

This cubic equation is satisfied for the value

$$a=3.5. \qquad \ldots \quad (80)$$

Adopting this value of a, (32) gives

$$V=\frac{ah^2\lambda}{4\pi^2 M}=7.7\times 10^{-18}. \qquad \ldots \quad (81)$$

From (81) and (24),

$$\lambda.V.=2.60\times 10^{-5} \text{ erg}=16.4 \text{ million electron-volts.} \qquad \ldots \quad (82)$$

The essence of the exchange theory is that two of the constituent particles of the nucleus virtually exchange their positions, and this is effected by the exchange particle transferring itself periodically from one of them to the other. The aid of a similar mechanism can be invoked to form a basis of the hypothesis advanced here that the rare gas nuclei form the principal constituents of the other nuclei. Thus a nucleus will consist of two equal particles of equal mass at equal distances and in line with a residual particle of the remainder of the mass of the nucleus. If this residual particle be taken at the centre of the co-ordinates, the co-ordinates of the two equal particles will be taken to be x and $-x$ respectively. According to our view, each of these equal terminal particles will be one of the rare gas nuclei composing the nucleus. Thus, Na^{23} nucleus will consist of two particles of protonic mass with a residual particle of mass 21 between them (Fig. 2).

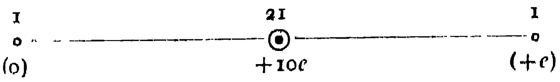

FIG. 2

The exchange particle will be a heavy positive electron oscillating between the extreme particles and converting each of them in turn from proton to neutron and back to proton, exactly as in deuteron. The central particle will then carry a charge $+10e$. Similarly the structure of Mg^{26} nucleus would be represented by Fig. 3.

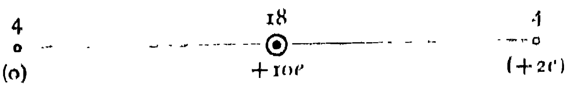

FIG. 3

The moments of inertia of some nuclei are known. For example,[8] the moment of inertia of B^{10} is 1.2×10^{-48}; of Mg^{26}, 6.25×10^{-48}; and of AcX^{223}, 2.2×10^{-46}. If I represents the moment of inertia and A the atomic number, then, the relation between log I and log A is very truly linear for the above three values, indicating the relation $I=2.53\times 10^{-50}\times A^{1.676}$. Thus, for Ba^{138},

$I = 1.0 \times 10^{-46}$. Presuming the structure of the Ba^{138} nucleus to be represented by Fig. 4, the distance between the extreme particles would be given by 1.0×10^{-46}

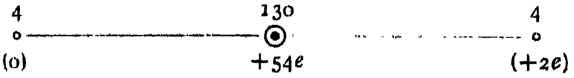

FIG. 4

$= 8 \times 1.67 \times 10^{-24} \times r^2$; or, $2r = 5.47 \times 10^{-12}$ cm. Similarly, for, Mg^{26}, $2r = 1.37 \times 10^{-12}$ cm. Taking these distances as the range, $1/\lambda$, of the interaction forces, we can calculate the mass, M, of the exchange particle from

$$M = \frac{h\lambda}{2\pi c}.$$

For Ba^{138}, $M = 6.35 \times 10^{-27}$ gm. $= 7.01 \times m$, and for Mg^{26}, $M = 2.535 \times 10^{-26}$ gm. $= 28.05 \times m$, where m is the electronic mass. We may, therefore, expect to find particles of masses $7 \times m$ and $28 \times m$ and of charge $+2e$ in any process involving the disintegration of the Mg^{26} and Ba^{138} nuclei.

We have assumed the constituents of the Mg^{26} nucleus to be Ne^{22} and He^4; and, therefore, the binding energy of Mg^{26} would be given by the mass defect of the constituent particles, and would amount to 1.91×10^{-5} erg. Applying as in (79), with $M = 6.68 \times 10^{-24}$ gm., $ab = 220.2$, and $a = 31.7$. Then, (32) would give $V = 3.8 \times 10^{-18}$. Similarly, for Ba^{138}, $a = 129$, and $V = 3.8 \times 10^{-18}$. Table II shows similar calculations for some other nuclei.

TABLE II

	$E \times 10^5$ ergs	$1/\lambda \times 10^{12}$ cm.	$I \times 10^{48}$ gm. cm.^{2}	M	e
Na^{23}	1.603	2.41	4.85	15.9	1
K^{39}	1.115	3.75	11 72	10.25	1
Cs^{133}	0.601	10.48	91.67	3.66	1
Mg^{24}	1.545	1.248	5.20	30.8	2
Mg^{25}	1.467	1.29	5 56	29.05	2
Mg^{26}	1.912	1 368	6.25	28.05	2
Ca^{40}	1.184	1.913	12.2	20.08	2
Ba^{138}	2.254	5 47	100 0	7.01	2
Cl^{35}	2.65	3.443	9.91	11.16	1
Cl^{37}	1.236	3.58	10.72	10.72	1
Br^{79}	0.75	3 30	38.3	5.67	1
Br^{81}	1.212	6 90	39.8	5 56	1
I^{127}	0 301	10.1	84.9	3.80	1
Ni^{58}	4.32	1 17	22.8	26.08	10
D	0.3421[11]	0.3833	—	130	1

The mass M of the exchange particle is represented in the above table in terms of the electronic mass as the unit.

E is calculated from the isotopic masses as given by Barkas[12] and by Aston.[13]

e is the charge upon the exchange particle in terms of the electronic charge as the unit.

The artificially activated fission of the uranium nucleus presents a problem bearing very directly on the view put forward here.[9] The products of this fission are known to be X, Kr ; Ba, Te ; Cs, I ; Br, Rb, Sr, Se ; La, Y ; Mo; Ma ; Sb, and, perhaps, Ag. It is of extreme interest to note that, excepting the doubtful product Ag, all the above-mentioned products of the fission are represented in the scheme of Table I as made up of either X or Kr as one of the components with the addition of a proton or He or Ne nucleus ; while the structure ascribed to the U nucleus is X plus Kr. Thus, Ba and Te are products from X and He nuclei ; I and Cs from X and H nuclei ; La and Y from X and Ne nuclei ; Br and Rb from Kr and H nuclei ; Se and Sr from Kr and He nuclei ; and Mo and Ma from Kr and Ne nuclei ; and Sb from Kr and A nuclei. This would indicate that the products of the fission are X, Kr, Ne, He, and protons, in addition to neutrons which have been observed experimentally. If we consider the U nucleus to be a binary system whose component are X and Kr, then, the lighter nuclei, He. Ne, and H, would have to be traced to the disruption of one or other of the X and Kr nuclei. The fission of uranium seems to provide a convincing demonstration of the fact that the constituents of the atomic nuclei are of comparable masses, and that these constituents are provided by the nuclei of the rare gases as outlined here.

For the time being, we can only speculate on the nature of the exact mechanism by which the incident neutrons loosen the bonds between the constituent particles sufficiently enough by disturbing the exchange forces to cause these constituents to fall asunder. It is probable that some kind of resonance is set up in the nucleus at the entry of the neutron into it.

Another aspect of our theory is the change in the isotopic number which would give a family of similar elements. In the original exposition, this was provided for on the assumption that, after the formation of the binary or multiple nucleus, the isotopic number was changing by absorption of negative electrons. For example, the synthesis of X and A produces nuclei of the second group of the rare earths, whose number is further increased by absorption of one or more electrons. We would then have elements ranging in atomic weights between 160 and 176, and ranging in atomic numbers from 72 downwards. This shows a fair agreement with facts, the atomic weights of the second group of the rare earths from Tb to Hf lying between 159 and 176 and the atomic numbers between 72 and 65. A better understanding of the phenomena leads us to believe that it is extremely improbable that electrons hold a free existence in nuclei. We have, on the other hand, abundant evidence to support the view that it is protons and neutrons that exist in the nuclei as their ultimate constituents. We can, therefore, replace the above assumption of the absorption of electrons by one of emission of positrons. That this is possible can be concluded from a comparison of the masses of the several primary particles involved in the process.[10] The masses of proton, neutron and electron are, respectively, 1.00812, 1.00893 and

0.00055 mass units. If we consider the neutron as the primary particle, then the proton consists of a neutron plus a positron, and its binding energy will be given by the mass defect

$$1.00893 + 0.00055 - 1.00812 = 0.00136 \text{ mass units.}$$

Evidently, this is not possible on the view that it is the proton that is the primary particle, and that the neutron is composed of a proton plus an electron. For, the binding energy would have to be

$$1.00812 + 0.00055 - 1.00893 = -0.00026 \text{ mass units,}$$

which is a negative quantity. Thus, there arises a probability of the proton disrupting into a neutron and a positron. The exact circumstances under which this would be possible cannot be understood ; but we have a similar case of positron emission in artificial radioactivity. Similarly induced instability in neuclei of binary or multiple character would change the isotopic number to yield the various elements of a group.

TABLE III

Isotope and their nuclear spins, I

Those for the rare gases are derived from the values of the others

Isotope and components	I
$Na^{23} = Ne^{22} + H$	3/2
$K^{39, 41} = A^{38, 40} + H$	3/2
$Br^{79} = Kr^{78} + H$	3/2
$Br^{81} = Kr^{80} + H$	3/2
$Rb^{85} = Kr^{84} + H$	5/2
$Rb^{87} = Kr^{86} + H$	3/2
$Cs^{133} = X^{129} + He$	7/2
$Eu^{151} = X^{(129, 131)} + Ne^{(22, 20)}$	5/2
H	1/2
He	0
Ne^{22} (20)	1
A^{38}	1
A^{40}	1
$Kr^{78, 80, 84, 86}$	1/2
$X^{129, (131)}$	7/2

We may be justified in assuming that the spin of a nucleus is the algebraic sum of the spins of its constituents. In Table III are listed some of the values of nuclear spins that are known with any degree of certainty. From these we can surmise the values of the spins of the constituents, which are also included in the table.

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